University of Diyala
College of Engineering
Department of Materials

Fundamentals of Electric Circuits

Lecture three

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2016

2-1 Resistance

It may be defined as the property of a substance due to which it opposes (or restricts) the flow of electricity (*i.e.*, electrons) through it.

The unit of measurement of resistance is the **ohm,** for which the symbol is Ω, the capital Greek letter omega. The resistance R offered by a conductor depends on the following factor:

(*i*) It varies directly as its length, l.

(*ii*) It varies inversely as the cross-section A of the conductor.

(*iii*) It depends on the nature of the material.

(*iv*) It also depends on the temperature of the conductor.

Neglecting the last factor for the time being, we can say that

$$R \propto \frac{l}{A} \quad \text{or} \quad R = \rho \frac{l}{A}$$

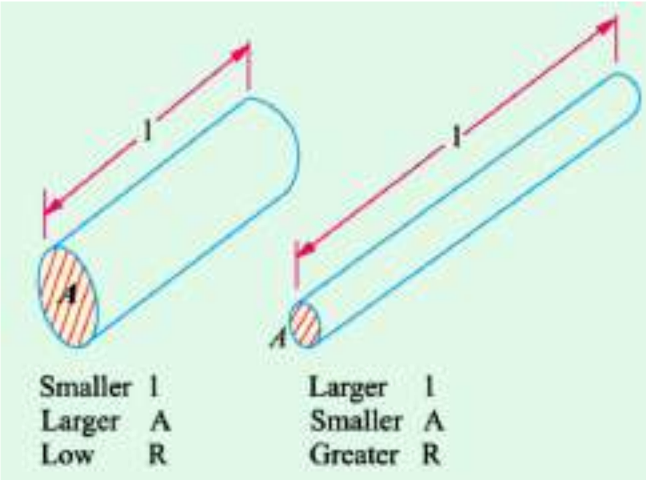


Fig. 2-1 Resistor

2-2 Units of Resistivity

In the S.I. system of units,

$$\rho = \frac{A \text{ metre}^2 \times R \text{ ohm}}{l \text{ metre}} = \frac{AR}{l} \text{ ohm-metre}$$

Hence, the unit of resistivity is ohm-metre (Ω-m).

It may, however, be noted that resistivity is sometimes expressed as so many ohm per m^3. Although, it is incorrect to say so but it means the same thing as ohm-metre.

If l is in centimetres and A in cm^2, then ρ is in ohm-centimetre (Ω-cm).

Values of resistivity and temperature coefficients for various materials are given in Table 1.3.

The resistivities of commercial materials may differ by several per cent due to impurities etc.

Material	*Resistivity in ohm-metre at 20°C (× 10^{-8})*	*Temperature coefficient at 20°C (× 10^{-4})*
Aluminium, commercial	2.8	40.3
Brass	6 – 8	20
Carbon	3000 – 7000	–5
Constantan or Eureka	49	+0.1 to -0.4
Copper (annealed)	1.72	39.3
German Silver	20.2	2.7
(84% Cu; 12% Ni; 4% Zn)		
Gold	2.44	36.5
Iron	9.8	65
Manganin	44 – 48	0.15
(84% Cu ; 12% Mn ; 4% Ni)		
Mercury	95.8	8.9
Nichrome	108.5	1.5
(60% Cu ; 25% Fe ; 15% Cr)		
Nickel	7.8	54
Platinum	9 – 15.5	36.7
Silver	1.64	38
Tungsten	5.5	47
Amber	5×10^{14}	
Bakelite	10^{10}	
Glass	$10^{10} – 10^{12}$	
Mica	10^{15}	
Rubber	10^{16}	
Shellac	10^{14}	
Sulphur	10^{15}	

Table 2-1

Example 1 / A coil consists of 2000 turns of copper wire having a cross-sectional area of 0.8 mm^2. The mean length per turn is 80 cm and the resistivity of copper is 0.02 μΩ–m. Find the resistance of the coil and power absorbed by the coil when connected across 110 $V_{d.c.}$ supply.

Solution. Length of the coil, $l = 0.8 \times 2000 = 1600$ m ; $A = 0.8 \text{ mm}^2 = 0.8 \times 10^{-6} \text{ m}^2$.

$$R = \rho \frac{l}{A} = 0.02 \times 10^{-6} \times 1600/0.8 \times 10^{-6} = \mathbf{40\ \Omega}$$

$$\text{Power absorbed} = V^2 / R = 110^2/40 = \mathbf{302.5\ W}$$

Example 2 / An aluminium wire 7.5 m long is connected in a parallel with a copper wire 6 m long. When a current of 5 A is passed through the combination, it is found that the current in the aluminium wire is 3 A. The diameter of the aluminium wire is 1 mm. Determine the diameter of the copper wire. Resistivity of copper is 0.017 μΩ-m ; that of the aluminium is 0.028 μΩ-m.

Solution:

Let the subscript 1 represent aluminium and subscript 2 represent copper.

$$R_1 = \rho \frac{l_1}{a_1} \text{ and } R_2 = \rho_2 \frac{l_2}{a_2} \qquad \therefore \frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} \cdot \frac{l_2}{l_1} \cdot \frac{a_1}{a_2}$$

$$\therefore \quad a_2 = a_1 \cdot \frac{R_1}{R_2} \cdot \frac{\rho_2}{\rho_1} \cdot \frac{l_2}{l_1} \qquad \text{...(i)}$$

Now $\quad I_1 = 3 \text{ A} ; I_2 = 5 - 3 = 2 \text{ A}.$

If V is the common voltage across the parallel combination of aluminium and copper wires, then

$$V = I_1 R_1 = I_2 R_2 \quad \therefore \quad R_1/R_2 = I_2/I_1 = 2/3$$

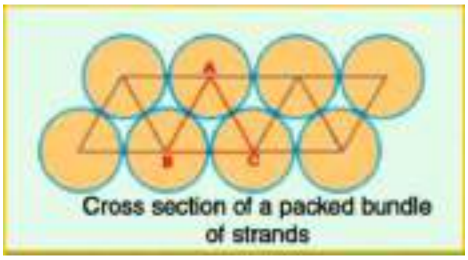
$$a_1 = \frac{\pi d^2}{4} = \frac{\pi \times 1^2}{4} = \frac{\pi}{4} \text{ mm}^2$$

Substituting the given values in Eq. (*i*), we get

$$a_2 = \frac{\pi}{4} \times \frac{2}{3} \times \frac{0.017}{0.028} \times \frac{6}{7.5} = 0.2544 \text{ m}^2$$

$$\therefore \quad \pi \times d_2^2/4 = 0.2544 \quad \text{or} \quad d_2 = \mathbf{0.569\ mm}$$

Example 3 / Calculate the resistance of 1 km long cable composed of 19 strands of similar copper conductors, each strand being 1.32 mm in diameter. Allow 5% increase in length for the 'lay' (twist) of each strand in completed cable. Resistivity of copper may be taken as 1.72 × 10−8 Ω m.



Solution. Allowing for twist, the length of the stands.

$$= 1000 \text{ m} + 5\% \text{ of } 1000 \text{ m} = 1050 \text{ m}$$

Area of cross-section of 19 strands of copper conductors is

$$19 \times \pi \times d^2/4 = 19\,\pi \times (1.32 \times 10^{-3})^2/4 \text{ m}^2$$

Now,

$$R = \rho\frac{l}{A} = \frac{1.72 \times 10^{-8} \times 1050 \times 4}{19\pi \times 1.32^2 \times 10^{-6}} = \mathbf{0.694\ \Omega}$$

Example 4 / The resistivity of a ferric-chromium-aluminium alloy is 51 × 10^−8 Ω-m. A sheet of the material is 15 cm long, 6 cm wide and 0.014 cm thick. Determine resistance between (a) opposite ends and (b) opposite sides

Solution. (*a*) As seen from Fig. 1. **4**(*a*) in this case,

$$l = 15 \text{ cm} = 0.15 \text{ cm}$$

$$A = 6 \times 0.014 = 0.084 \text{ cm}^2$$

$$= 0.084 \times 10^{-4} \text{ m}^2$$

$$R = \rho\frac{l}{A} = \frac{51 \times 10^{-8} \times 0.15}{0.084 \times 10^{-4}}$$

$$= \mathbf{9.1 \times 10^{-3}\ \Omega}$$

(*b*) As seen from Fig. 1.**4** (*b*) here

$$l = 0.014 \text{ cm} = 14 \times 10^{-5} \text{ m}$$

$$A = 15 \times 6 = 90 \text{ cm}^2 = 9 \times 10^{-3} \text{ m}^2$$

$$R = 51 \times 10^{-8} \times 14 \times 10^{-5}/9 \times 10^{-3} = \mathbf{79.3 \times 10^{-10}\ \Omega}$$

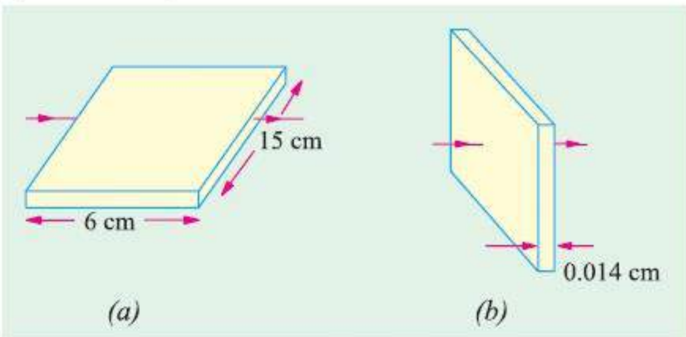


Fig. 1.4

Example 5 / A piece of silver wire has a resistance of 1 Ω. What will be the resistance of manganin wire of one-third the length and one-third the diameter, if the specific resistance of manganin is 30 times that of silver.

Solution. For silver wire, $R_1 = \frac{l_1}{A_1}$; For manganin wire, $R = \rho_2 \frac{l_2}{A_2}$

$$\therefore \quad \frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} \times \frac{l_2}{l_1} \times \frac{A_1}{A_2}$$

Now $\quad A_1 = \pi d_1^2/4 \quad$ and $\quad A_2 = \pi d_2^2/4 \qquad \therefore A_1/A_2 = d_1^2/d_2^2$

$$\therefore \quad \frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} \times \frac{l_2}{l_1} \times \left(\frac{d_1}{d_2}\right)^2$$

$$R_1 = 1\ \Omega;\ l_2/l_1 = 1/3,\ (d_1/d_2)^2 = (3/1)^2 = 9;\ \rho_2/\rho_1 = 30$$

$$\therefore \quad R_2 = 1 \times 30 \times (1/3) \times 9 = \mathbf{90\ \Omega}$$

H.W. 1

A cube of a material of side 1 cm has a resistance of 0.001 Ω between its opposite faces. If the same volume of the material has a length of 8 cm and a uniform cross-section, what will be the resistance of this length?

Answer: 0.064 Ω

H.W. 2

A rectangular metal strip has the following dimensions : x = 10 cm, y = 0.5 cm, z = 0.2 cm

Determine the ratio of resistances R_x, R_y, and R_z between the respective pairs of opposite faces.

Answer: [Rx : Ry : Rz :10000 : 25 : 4]